Model Solver

The optimization of **Subproblem 1** is our core of optimization. In the following, we describe it in detail. The minimization is defined as,

\[
\arg \min_G ||G||_0 + \mu^{k-1} ||\nabla R^{k-1} - G + X^{k-1}||^2_2.
\] (1)

The above problem can be solved in an element-wise manner. Formally, for a pixel \( p \), the solution is

\[
G^k_p = \begin{cases} 
0, & (Y^{k-1}_p)^2 \leq \frac{1}{\mu^{k-1}}, \\
Y^k_p, & \text{otherwise} 
\end{cases}
\] (2)

where \( Y^{k-1} = X^{k-1} - \nabla R^{k-1} \).

This solution can be proved as follows.

**Proof.** We use \( E_p \) to represent the energy of the \( j \)-th scalar function in Eq. (2), expressed as

\[
E_p = \frac{1}{\mu^{k-1}} ||G_p||_0 + (G_p - Y^{k-1}_p)^2. \] (3)

(1) When \((Y^{k-1}_p)^2 \leq \frac{1}{\mu^{k-1}}\), the energy for non-zero \( G_p \) is

\[
E_p(G_p \neq 0) = \frac{1}{\mu^{k-1}} + (G_p - Y^{k-1}_p)^2 \geq \frac{1}{\mu^{k-1}} \geq (Y^{k-1}_p)^2, \] (4)

and for the zero \( G_p \) is

\[
E_p(G_p = 0) = (Y^{k-1}_p)^2. \] (5)

Since \( E_p(G_p \neq 0) \geq (Y^{k-1}_p)^2 \geq E_p(G_p = 0) \), we have the solution: \( G_p = 0 \) when \((Y^{k-1}_p)^2 \leq \frac{1}{\mu^{k-1}}\).

(2) When \((Y^{k-1}_p)^2 > \frac{1}{\mu^{k-1}}\), Eq. (5) is still true. In addition, for non-zeros \( G_p \), \( E_p(G_p \neq 0) \) has a minimum \( \frac{1}{\mu^{k-1}} \) at \( G_p = Y^k_p \). Since

\[
E_p(G_p = Y^k_p) = \frac{1}{\mu^{k-1}} \leq (Y^{k-1}_p)^2 = E_p(G_p = 0), \] (6)

, the solution is \( G_p = Y^{k-1}_p \) when \((Y^{k-1}_p)^2 > \frac{1}{\mu^{k-1}}\). \qed
More Visual Results

Figure 1: Comparison of intrinsic image decomposition between our method and other state-of-the-art methods.
Figure 2: Comparison of intrinsic image decomposition between our method and other state-of-the-art methods.

Figure 3: Comparison of intrinsic image decomposition between our method and other state-of-the-art methods.
Figure 4: Comparison of intrinsic image decomposition between our method and other state-of-the-art methods.

Figure 5: Comparison of intrinsic image decomposition between our method and other state-of-the-art methods.
Figure 6: Comparison of intrinsic image decomposition between our method and other state-of-the-art methods.

Figure 7: Comparison of intrinsic image decomposition between our method and other state-of-the-art methods.
Figure 8: Comparison of intrinsic image decomposition between our method and other state-of-the-art methods.
References


